Last time: • Translation of statements

• Logical Connectives ⟷ logic operations

Negation:	$\neg A$	"Not A"
Conjunction:	$A \wedge B$	"Both A and B".
Disjunction:	$A \vee B$	"Either A or B".
Implication:	$A \to B$	"A implies B" or "If A, then B".
Biconditional:	$A \leftrightarrow B$	"A precisely when B" or "A if and only if B".

TRUTH TABLES

IDEA: Determine truth value of a compound statement from its atomic "building blocks".

Negation:

A	¬A
T	F
F	T

(¬A is true exactly when A is false)

Conjunction:

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

A	B	A ∨ B
T	T	T
T	F	T
F	T	T
F	F	F

Inclusive - OR

Biconditional

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Implication

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Ex: If you eat an apple in the morning, doctors will avoid you.

Ex: Construct a truth table for $(\neg P) \to Q$.

Remark: Parens are very important!

$\neg P \to Q$ is ambiguous! :(

Could interpret as either: $(\neg P) \to Q$ or $\neg(P \to Q)$

Sol:

P	Q	$\neg P$	$(\neg P) \to Q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

$F \to T \equiv T$

Ex: Build a truth table for $P \to ((\neg Q) \vee (R \leftrightarrow Q))$.

Sol:

P	Q	R	$\neg Q$	$R \leftrightarrow Q$	$(\neg Q) \vee (R \leftrightarrow Q)$	$P \to ((\neg Q) \vee (R \leftrightarrow Q))$
T	T	T	F	T	T	T
T	T	F	F	F	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	T	T	T

Q: When are two statements the same?

Ex: "It is not the case that a dragon eats meat and is vegan." $\neg(M \wedge V)$

"Either the dragon does not eat meat or the dragon is not vegan" $(\neg M) \vee (\neg V)$

Defn: Propositions $\mathbb{A}$ and $\mathbb{B}$ are logically equivalent when $\mathbb{A}$ and $\mathbb{B}$ have the same final column of their truth tables.

Ex: Show $\neg(M \wedge V)$ is logically equivalent to $(\neg M) \vee (\neg V)$.

Sol:

M	V	$M \wedge V$	$\neg(M \wedge V)$	$\neg M$	$\neg V$	$(\neg M) \vee (\neg V)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Same final columns

$\therefore$ $\neg(M \wedge V)$ is logically equivalent to $(\neg M) \vee (\neg V)$ by definition of logical equivalence.

Notation: We write $\mathbb{A} \equiv \mathbb{B}$ to denote statement $\mathbb{A}$ is logically equivalent to statement $\mathbb{B}$.

In this notation, we showed above $\neg(M \wedge V) \equiv (\neg M) \vee (\neg V)$.

Defn: A statement $\mathbb{A}$ is...

1. a tautology when $\mathbb{A}$ is true always.
2. a contradiction when $\mathbb{A}$ is always false.
3. conditionally true when $\mathbb{A}$ is neither a tautology nor a contradiction.
   ↳ i.e. there is an assignment of truth values producing T in the final column and another producing F.

Ex (verify these at home):

1. $P \vee (\neg P)$ is a tautology.
2. $P \wedge (\neg P)$ is a contradiction.
3. $P \to Q$ is conditionally true.

Basic Equivalences

Name	Formula
Double Negation	$\neg(\neg P) \equiv P$
De Morgan's Laws	$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$
	$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$
Material Implication	$P \to Q \equiv (\neg P) \vee Q$
Contraposition	$P \to Q \equiv (\neg Q) \to (\neg P)$
Biconditional Expansion	$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$
Commutativity	$P \vee Q \equiv Q \vee P$
	$P \wedge Q \equiv Q \wedge P$
Associativity	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
	$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
Distributivity	$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Tautology $P \vee (\neg P) \equiv T$

Contradiction $P \wedge (\neg P) \equiv F$

Simplification $P \wedge T \equiv P$ $\quad P \wedge F \equiv F$

$P \vee F \equiv P$ $\quad P \vee T \equiv T$

Homework: Verify all of these equivalences.